

Suppose we have  $n$  jobs  $J_1, \dots, J_n$  with run times  $t_1 \geq t_2 \geq \dots \geq t_n$  and  $k > 1$  machines  $M_1, \dots, M_k$ . Define:

$$A = \frac{1}{k} \sum_{i=1}^n t_i$$

If  $t_1 \geq A$ , then an optimal solution to the job makespan minimization problem can be obtained by assigning  $J_1$  to  $M_1$  and solving the makespan minimization subproblem for the  $n - 1$  jobs  $J_2, \dots, J_n$  with the  $k - 1$  machines  $M_2, \dots, M_k$ .

*Proof:* Suppose for contradiction that this algorithm did not give the optimal solution to the makespan minimization problem. Then, since it was given that the subproblem was optimal, it must be the case that in the optimal solution,  $M_1$  must run another job  $J_j$ . Since  $t_1 \geq A$ , and  $M_1$  runs both jobs  $J_1$  and  $J_j$ ,  $M_1$  has a run time strictly greater than  $A$ . But, since  $A$  is the average runtime for the machines, it then follows from this that some other machine  $M_l$  must have a run time strictly less than  $A$ . If we assign  $J_j$  instead to  $M_l$ , then the makespan of  $M_1$  and  $M_l$  strictly decreases. This gives at least an equally valid solution (if not strictly better). In this way, we can reassign any other job assigned to  $M_1$  (other than  $J_1$ ) and still have an optimal solution, which is a contradiction.